Grade 9
Surface Area and Volume

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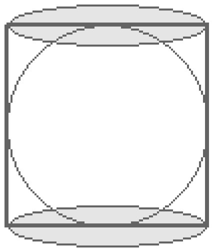
Choose correct answer(s) from the given choices

(1) A sphere is just enclosed inside a right circular cylinder. If the volume of the gap between the cylinder and the sphere is 10 cm^3, find the volume of the sphere.



a. 10 cm^3 **b.** 25 cm^3
c. 40 cm^3 **d.** 20 cm^3

(2) The radius of a cylinder is doubled and the height is tripled. What is the area of the curved surface now compared to the previous surface area?

a. 3 times **b.** 4 times
c. 5 times **d.** 6 times

(3) If the radii of two hemispheres are in ratio 1:5, find the ratio of their surface area.

a. 5:1 **b.** 25:1
c. 125:1 **d.** 1:25

(4) If radius of a hemisphere is 2a, find its volume.

a. $\frac{128}{3}\pi a^3$ **b.** $\frac{16}{3}\pi a^3$
c. $\frac{2}{3}\pi a^3$ **d.** $\frac{54}{3}\pi a^3$

(5) The area of a trapezium is 315 cm^2 and the distance between its parallel sides is 15 cm. If one of the parallel sides is of length 24 cm, find the length of the other side.

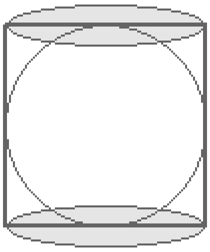
a. 9 cm **b.** 18 cm
c. 36 cm **d.** 16 cm

Fill in the blanks

(6) The area of a trapezium is 204 cm^2 and the distance between its parallel sides is 12 cm. If one of the parallel sides is of length 22 cm, the length of the other side is ____ cm.

Answer the questions

(7) A sphere is just enclosed inside a right circular cylinder. If the total surface area of the cylinder is 165 cm^2, find the surface area of the sphere.



(8) The area of a trapezium is 243 cm^2 and the distance between its parallel sides is 27 cm. If one of the parallel sides is of length 12 cm, find the length of the other side.

(9) Find the surface area of the biggest sphere which can fit inside a cube of side 6b.

(10) A sphere and a right circular cylinder have the same radius. If the volume of the sphere is triple of the volume of the cylinder, what is the ratio of the height of the cylinder to its radius?

Solutions

(1) **d.** 20 cm^3

Step 1

There are three equations we need to know in this type of question - the volume of a cylinder, the volume a sphere, and the remaining volume of the gap between the sphere and the cylinder.

Step 2

The volume of a cylinder of radius 'r' and height 'h' is $\pi r^2 h$. Here, we know the sphere will fit in exactly in the cylinder, so h=2r, and the formula now becomes $2\pi r^3$.

Step 3

The sphere will have the radius 'r' so its volume is $\frac{4}{3} \pi r^3$.

Step 4

The volume of the gap between the cylinder and the sphere is all the volume inside the cylinder not taken up by the sphere.
This is the difference between the volume of the cylinder and the volume of the sphere.

i.e. volume of the gap = $2\pi r^3 - \frac{4}{3} \pi r^3$

Simplifying, volume of the gap = $\frac{2}{3} \pi r^3$

Step 5

So we have 3 equations:

Volume of the cylinder = $2\pi r^3$

Volume of the sphere = $\frac{4}{3} \pi r^3$

Volume of the gap = $\frac{2}{3} \pi r^3$

Step 6

Here, we know that volume of the gap between the cylinder and the sphere is 10 cm^3. We need to find the volume of the sphere.

Step 7

Substituting from the equation above, we get volume of the sphere = **20 cm^3**

(2) **d.** 6 times

Step 1

The curved surface area of a cylinder is $2\pi rh$.

Step 2

If the radius is doubled and the height is tripled.

Step 3

Putting this into the formula, we see that the curved surface area becomes:
$2\pi \times 2 \times r \times 3 \times h$
$= 6(2\pi rh)$
= 6 times.

(3) **d.** 1:25

Step 1

The surface area of a sphere of radius x is given by $4\pi x^2$.

Step 2

The surface area of a hemisphere is $3\pi x^2$ (half of the surface area of the sphere, plus the area of the base, which is a circle of radius x.

Step 3

Assume the radii of these two hemispheres are x and $5x$. Note that this allows us to get the ratio of 1:5, which is the only thing we know about these radii.

Step 4

The surface area of the first hemisphere will become $3\pi x^2$ and the surface area of the second hemisphere will become $3\pi \times (5x)^2 = 75\pi x^2$.

Step 5

Hence, we can see that the ratio of the surface area of two given hemispheres is $3\pi \times x^2$: $75\pi x^2$
= 3 : 75
= **1:25**

(4) **b.** $\frac{16}{3}\ \pi a^3$

Step 1

The volume of a hemisphere of radius *x* is given by $\frac{4}{3}\ \pi x^3$.

Step 2

The volume of a hemisphere is half that i.e. $\frac{2}{3}\ \pi x^3$.

Step 3

The above expression is valid when the radius is x. We will have to replace it by 2a as per the question:

Volume of hemisphere = $\frac{2}{3}\ \pi \times (2a)^3$

$= \frac{2}{3}\ \pi \times 8a^3$

$= \frac{16}{3}\ \pi a^3$.

Step 4

This gives us the answer as $\frac{16}{3}$ $\boldsymbol{\pi a^3}$.

(5) **b.** 18 cm

Step 1

We have been told:

Area of the trapezium (A) = 315 cm^2
Distance between its parallel sides (d) = 15 cm
Length of one parallel side (a) = 24 cm

Step 2

We know that:

Area of a trapezium = $\frac{1}{2}$ (a + b) d,

where a and b are the lengths of the two parallel sides and d is the distance between them.

Step 3

Let's substitute the known values into the formula, and solve for b:

$$315 = \frac{1}{2} \times (24 + b) \times 15$$

$$\Rightarrow \frac{315 \times 2}{15} = (24 + b)$$

$$\Rightarrow b = 42 - 24$$

$$\Rightarrow b = 18$$

Step 4

Therefore, the length of the other side is **18 cm**.

(6) 12

Step 1

We have been told:

Area of the trapezium (A) = 204 cm^2
Distance between its parallel sides (d) = 12 cm
Length of one parallel side (a) = 22 cm

Step 2

We know that:

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)\,d,$$

where a and b are the lengths of the two parallel sides and d is the distance between them.

Step 3

Let's substitute the known values into the formula, and solve for b:

$$204 = \frac{1}{2} \times (22 + b) \times 12$$

$$\Rightarrow \quad \frac{204 \times 2}{12} = (22 + b)$$

$$\Rightarrow \quad b = 34 - 22$$

$$\Rightarrow \quad b = 12$$

Step 4

Therefore, the length of the other side is **12 cm**.

(7) 110 cm^2

Step 1

There are three equations we need to know in this type of question - the total surface area of a cylinder, the curved surface area of a cylinder, and the surface area of a sphere.

Step 2

The curved surface area of a cylinder of radius 'r' and height 'h' is $2\pi rh$. Here we know the sphere is enclosed in the cylinder, so h=2r, and the formula now becomes $4\pi r^2$.

Step 3

The total surface area of the same cylinder will be the sum of the curved area and the surface area of the two circles at top and bottom. So, total surface area of the cylinder = $4\pi r^2 + 2\pi r^2 = 6\pi r^2$

Step 4

And of course, the sphere will have the radius 'r'. So, its surface area is $4\pi r^2$.

Step 5

From these equations, we see that for this case, the surface area of the sphere is the same as the curved surface area of the cylinder, and $\frac{2}{3}$ of the total surface area of the cylinder.

Step 6

Here, we know that the total surface area of the cylinder is 165 cm^2. We need to find the surface area of the sphere.

Step 7

Using step 5, we get the surface area of the sphere = **110 cm^2**

(8) 6 cm

Step 1

We have been told:

Area of the trapezium (A) = 243 cm^2
Distance between its parallel sides (d) = 27 cm
Length of one parallel side (a) = 12 cm

Step 2

We know that:

Area of a trapezium = $\frac{1}{2}$ $(a + b)$ d,

where a and b are the lengths of the two parallel sides and d is the distance between them.

Step 3

Let's substitute the known values into the formula, and solve for b:

$$243 = \frac{1}{2} \times (12 + b) \times 27$$

$$\Rightarrow \frac{243 \times 2}{27} = (12 + b)$$

$$\Rightarrow b = 18 - 12$$

$$\Rightarrow b = 6$$

Step 4

Therefore, the length of the other side is **6 cm**.

(9) $36\pi b^2$

Step 1

The biggest sphere that can fit inside a cube of side 6b will have a diameter of 6b (anything larger will not fit in, as opposite sides are separated by a distance of 6b).

Step 2

This means that the radius of this sphere is $\frac{1}{2} \times 6b = 3b$.

Step 3

We know that the surface area of a sphere of radius x is $4\pi x^2$.

So, the surface area of the given sphere of radius 3b is $4\pi(3b)^2$

$= 4\pi \times 9b^2$

$= \mathbf{36\pi b^2}$

(10) 4:9

Step 1

The volume of a sphere of radius 'r' = $\frac{4}{3} \pi r^3$

Step 2

The volume of a cylinder of radius 'r' and height 'h' = $\pi r^2 h$

Step 3

Here, we are told the the volume of the sphere is triple of the volume of the cylinder.

So, $\frac{4}{3} \pi r^3 = 3 \times (\pi r^2 h)$

Step 4

Solving the above equation, we get 9h = 4r.

Step 5

Therefore, the ratio of the height of the cylinder to its radius is 4:9.